LTE Downlink Radio Scheduler - Analytical Modeling

FFV, Aachen - March 16th, 2012

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Introduction

Problem Statement
LTE Schedulers

General Analytical Model

Assumptions
General Analytical Model
Avg. Bit Rate $\overline{TBS}(n)$

Single Class Model

Single Class
Validation

Two Classes Model

Two Classes
Validation

Conclusion
Problem Statement

- This work targets the analytical modeling of the LTE MAC scheduler
- The presented model is an extension of the model in [1]
- The scheduler has a fixed amount of Physical Resource Blocks (PRBs)
- The scheduler distributes the PRBs among the active users every TTI \(^1\)
- The UEs have different SINR i.e. can support different Modulation and Coding Scheme (MCS)


\(^1\)Transmission Time Interval = 1ms
LTE Time Domain Schedulers

MaxT: Maximum Throughput

\[ P_k(t)_{\text{MaxT}} = \arg\max_k [\text{SINR}_k[t]] \]

where \( P_k(t)_{\text{MaxT}} \) is the time domain priority factor, \( k \) is the user number and \( \text{SINR}_k[t] \) is the instantaneous SINR of user \( k \).

BET: Blind Equal Throughput

\[ P_k(t)_{\text{BET}} = \arg\max_k \left[ \frac{1}{\bar{\theta}_k[t]} \right] \]

where \( \bar{\theta}_k[t] \) is the normalized average throughput of user \( k \).
LTE Frequency Domain Scheduler

- The FDS is round robin scheduler
- It serves up to $\psi=5$ users per TTI
- This means, that the UEs served per TTI are equal to $\eta$:

$$\eta = \min(n - n_0, \psi)$$

with $n$ being the number of active users in a TTI, and $n_0$ is the number of users in outage
Model Assumptions

- A UE can use the whole spectrum
- UEs are configured with only one bearer
- UEs are statistically identical with respect to the channel
- The channel is divided into 8 states ($MCS_k$) ($0 < k < 7$)
- Each $MCS_k$ has a static probability $P_k$
- UEs use the same ON/OFF elastic traffic:
  - $\overline{X_{ON}}$ is the average file size in bits
  - $\overline{t_{off}}$ is the average OFF duration in s

<table>
<thead>
<tr>
<th>MCS</th>
<th>Probability $P_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0.15</td>
</tr>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
</tr>
<tr>
<td>5</td>
<td>0.3</td>
</tr>
<tr>
<td>6</td>
<td>0.35</td>
</tr>
<tr>
<td>7</td>
<td>0.4</td>
</tr>
</tbody>
</table>

[Graph showing probability distribution of MCS]
General Analytical Model

- The model is based on the Time Contiguous Markov Chain with a state n representing the number of active UEs in a TTI
- The resulting CTMC is made of N+1 states
  - Arrival transition is performed with rate \((N - n)\lambda\) with \(\lambda = 1/t_{off}\)
  - Departure transition is performed with a generic rate \(\mu(n)\)
Steady State Probability

- Provided that $\mu(n)$ can be estimated, the steady state probability $\pi(n)$ can be calculated from:

$$\pi \cdot Q = 0$$  \hspace{1cm} (1)

where $\pi$ is a vector representing the steady state probability, i.e, $\pi=[\pi_0, \pi_1, \ldots \pi_N]$ and $\sum_i \pi_i = 1$

- $Q$ is the infinitesimal generator matrix.
Performance Parameters

- The average number of active users $\overline{Q}$ is expressed as:
  \[
  \overline{Q} = \sum_{n=1}^{N} n\pi(n)
  \]

- The mean number of departures by unit of time is $\overline{D}$ and is obtained as:
  \[
  \overline{D} = \sum_{n=1}^{N} \mu(n)\pi(n)
  \]

- According to Little’s law, the average ON period duration $\overline{t_{ON}}$ (i.e. duration of an active transfer):
  \[
  \overline{t_{on}} = \frac{\overline{Q}}{\overline{D}}
  \]
Departure Rate $\mu(n)$

$$
\mu(n) = \frac{TBS(n)}{X_{on} \cdot TTI}
$$

- $\overline{TBS(n)}$ is the average amount of bits sent by all served users within a TTI
- $\overline{X_{on}}$ is the average file size of the ON period
- TTI is the Transmission Time Interval (1 ms in LTE)
Generic Average Bit Rate $\overline{TBS}(n)$

$$\overline{TBS}(n) = \sum_{(n_0,...,n_K)=(0,...,0)}^{(n,...,n)} \overline{TBS}(n_0,...,n_K) \binom{n}{n_0,...,n_K} \prod_{k=0}^{K} P_k^{n_k}$$

- $\binom{n}{n_0,...,n_K}$ is the multinomial coefficient
- $P_k$ is the stationary probability of $MCS_k$
- $n_k$ is the number of active users in $MCS_k$
- $\overline{TBS}(n_0,...,n_K)$ is the total bits transmitted for all served users under a certain combination $(n_0,...,n_K)$
\( \overline{TBS}(n_0, \ldots, n_K) \)

\[
\overline{TBS}(n_0, \ldots, n_K) = \sum_{i=[i_1, \ldots, i_\eta]} TBS_i(\eta) \quad (4)
\]

- \( \eta \) is the number of users served under \((n_0, \ldots, n_K)\)
- \([i_1, \ldots, i_\eta]\) represents index of the chosen \(\eta\) users out of the \(n\) active users considered for scheduling within a TTI
- \(TBS_i(\eta)\) is transmitted bits of user \(i\) using his MCS (when \(\eta\) users are served)
- the choice of the \(\eta\) users is determined by the TD scheduler.
Chosen $\eta$ UEs Procedure

- For the MaxT scheduler the $\eta$ chosen users are the ones with the highest MCS, and represent the $i$ users indices ($[i_1, \ldots, i_\eta]$).

- For the BET scheduler the $\eta$ users are chosen as follows:

\[
\begin{array}{cccccc}
\text{MCS}_0 & \text{MCS}_1 & \text{MCS}_2 & \text{MCS}_3 & \text{MCS}_4 & \text{MCS}_5 \\
 n_0 & n_1 & n_2 & n_3 & n_4 & n_5 \\
\end{array}
\quad
\begin{array}{cccc}
\text{MCS}_6 & \text{MCS}_7 \\
 n_6 & n_7 \\
\end{array}
\]

Find $\eta$ UEs

\[
\begin{array}{cccccc}
\text{MCS}_0 & \text{MCS}_1 & \text{MCS}_2 & \text{MCS}_3 & \text{MCS}_4 & \text{MCS}_5 \\
 n_0 & n_1 & n_2 & n_3 & n_4 & n_5 \\
\end{array}
\quad
\begin{array}{ccc}
\text{MCS}_6 & \text{MCS}_7 \\
 n_6 & n_7 \\
\end{array}
\]

Find ($\eta-1$) UEs

Find 1 UE
Scenarios Configuration

- The results of the analytical model is compared against the OPNET simulation model

<table>
<thead>
<tr>
<th>General parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth (# PRBs)</td>
<td>5 MHz (i.e., 25 PRBs)</td>
</tr>
<tr>
<td>Traffic model</td>
<td>FTP with different IATs and file sizes</td>
</tr>
<tr>
<td>Simulation run time</td>
<td>8000 s, 10 seeds with 95% confidence interval</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Scheduler</th>
<th># of UEs</th>
<th>FTP file size</th>
<th>FTP IAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis1</td>
<td>MaxT</td>
<td>10 UEs</td>
<td>constant 5,10 and 15 MB</td>
<td>uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s &amp; uniform (45, 75) s</td>
</tr>
<tr>
<td>Analysis2</td>
<td>BET</td>
<td>10 UEs</td>
<td>constant 5,10 and 15 MB</td>
<td>uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s &amp; uniform (45, 75) s</td>
</tr>
</tbody>
</table>
Average FTP Download Time

MaxT Average FTP download time (10UEs scenario)

BET Average FTP download time (10UEs scenario)

MaxT Markov Chain State Probability (10UEs 5MB scenario)

BET Markov Chain State Probability (10UEs 5MB scenario)
Two Classes Model

- The single class model is extended into a two dimensional Markov Chain, with state (i,j)
  - i represents the number of UEs within class 1
  - j represents the number of UEs within class 2
- The combinations within each state will increase to:
  \[(i_0, i_1, i_2, i_3, i_4, i_5, i_6, j_0, j_1, j_2, j_3, j_4, j_5, j_6, j_7)\]
  where \(\sum_r i_r = N1\) and \(= \sum_r j_r = N2\)
Departure Rate $\mu_1(n)$

$$\mu_1(i, j) = \frac{TBS_1(i, j)}{X_{1on} \cdot TTI}$$

$$TBS_1(i, j) = \sum_{(i_0, \ldots, i_K, j_0, \ldots, j_K) = (0, \ldots, 0)}^{(i, \ldots, i, j, \ldots, j)} i!j! TBS_1(i_0, \ldots, i_K, j_0, \ldots, j_K) \left[ \prod_{k=0}^{K} \frac{P_{i_k} P_{j_k}}{i_k!j_k!} \right]$$

$$TBS_1(i_0, \ldots, i_K, j_0, \ldots, j_K) = \sum_{r = [r_1, \ldots, r_{\delta_1}]} TBS_r(\eta)$$

with $r_{\delta_1} + r_{\delta_2} = \eta$
# Scenarios Configuration

<table>
<thead>
<tr>
<th>General parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bandwidth (# PRBs)</td>
<td>5 MHz (i.e., 25 PRBs)</td>
</tr>
<tr>
<td>Radio QoS weights</td>
<td>$W_{QoS_1} = 2$ and $W_{QoS_2} = 1$</td>
</tr>
<tr>
<td>Number of users</td>
<td>9 UEs (3 in class1 &amp; 6 in class2)</td>
</tr>
<tr>
<td>Traffic model</td>
<td>FTP with different IATs and file sizes</td>
</tr>
<tr>
<td>Simulation run time</td>
<td>8000 s, 10 seeds with 95% confidence interval</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Analysis</th>
<th>C1 FTP file size</th>
<th>C2 FTP file size</th>
<th>FTP IAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analysis1</td>
<td>constant 5MByte</td>
<td>Constant 10 MByte</td>
<td>uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s &amp; uniform (45, 75) s</td>
</tr>
<tr>
<td>w-MaxT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Analysis2</td>
<td>constant 5MByte</td>
<td>Constant 10 MByte</td>
<td>uniform (0, 30) s, uniform (15, 45) s, uniform (30, 60) s &amp; uniform (45, 75) s</td>
</tr>
<tr>
<td>w-BET</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Average FTP Download Time

![Graph of Average FTP Download Time](image)

- **w−MaxT Average FTP download time**
- **w−BET Average FTP download time**

**Introduction**

**General Analytical Model**

**Single Class Model**

**Two Classes Model**

**Validation**

**Conclusion**

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Conclusion & Outlook

- Two different TDS (MaxT and BET) are modeled analytically for single class case
- The analytical model results match the simulation ones
- The w-MaxT and w-BET TDS are modeled with two classes and QoS differentiation
- Results of the 2D model match the simulation results
- The two classes model can be extended for other TDS (e.g., w-PF)
- Additional GBR class can also be modeled
Thanks for Listening
Any Question ??
BACKUP SLIDES
A transition from state $n \rightarrow n+1$ occurs when a UE in OFF period ends its reading time and initiate a data transfer:

- Arrival transition is performed with rate $(N - n)\lambda$ with $\lambda = \frac{1}{t_{\text{off}}}$

A transition from state $n \rightarrow n-1$ occurs when a UE in ON period complete its transfer:

- Departure transition is performed with a generic rate $\mu(n)$
- The difficulty lies in estimating the departure rate $\mu(n)$

\[
\begin{align*}
0 & \overset{N\lambda}{\longrightarrow} 1 \overset{(N-1)\lambda}{\longrightarrow} 2 \overset{(N-n+1)\lambda}{\longrightarrow} n-1 \overset{(N-n)\lambda}{\longrightarrow} n \overset{(N-n)\lambda}{\longrightarrow} n+1 \overset{(N-n)\lambda}{\longrightarrow} N \\
\mu(1) & \quad \mu(2) \quad \mu(n) \quad \mu(n+1) \quad \mu(N)
\end{align*}
\]
Packet Scheduling Framework

- Time Domain Scheduling (TD)
- Frequency Domain Scheduling (FD)
- Outer Loop Link Adaptation
- Inner Loop Link Adaptation
- MIMO Link Adaptation
- HARQ
- QoS attributes
- Buffer information
- Rank
- CQI
- (N)ACK
Channel Dependent Scheduling
LTE Radio Frame

- 1 LTE radio frame
- 1 sub-frame (1 ms)
- 1 slot (0.5 ms)
- 7 Symbols = 1 Resource Block = 1 slot

- Resource Block 0
- Resource Block 1
- Resource Block 2
- Resource Block 3
- Resource Block N

- N represents the maximum number of resource blocks which depends on the defined spectrum/bandwidth

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Steady State Probability

Another solution will be to use the closed loop solution from the birth-and-death process of the Markov chain:

\[
\pi(n) = \left[ \prod_{i=1}^{n} \frac{(N - i + 1)\lambda}{\mu(i)} \right] \pi(0)
\]

\(\pi(0)\) is obtained by normalization.
if we have two active UEs i.e. \( n=2 \)

those two users can be at any MCS (from \( MCS_0 \) to \( MCS_7 \))

if we represent the number of users in each \( MCS_k \) by \( n_k \)

we get: \( (n_0, n_1, n_2, n_3, n_4, n_5, n_6, n_7) \) with \( n_0 + ... + n_7 = n \)

then we have the following possible combinations

<table>
<thead>
<tr>
<th></th>
<th>MCS0</th>
<th>MCS1</th>
<th>MCS2</th>
<th>MCS3</th>
<th>MCS4</th>
<th>MCS5</th>
<th>MCS6</th>
<th>MCS7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comb1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Comb2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Comb3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
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<tr>
<td>Comb4</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Comb5</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Comb6</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Comb7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Comb8</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Comb9</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>1</td>
<td>0</td>
<td>0</td>
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<td>Comb10</td>
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<td>0</td>
<td>0</td>
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<td>Comb11</td>
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<td>1</td>
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<td>1</td>
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<tr>
<td>Comb12</td>
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<tr>
<td>Comb13</td>
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<td>0</td>
<td>1</td>
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</tr>
<tr>
<td>Comb14</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Comb15</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
\[
TBS(2) = \sum_{(n_0,\ldots,n_K)=(0,\ldots,0) | n_0 + \ldots + n_K = 0}^{(2,\ldots,2)} TBS(n_0,\ldots,n_K) \left( \begin{array}{c} 2 \\ n_0,\ldots,n_K \end{array} \right) \left[ \prod_{k=0}^{K} P_{n_k}^{n_k} \right]
\]

(5)

Average number of bits transmitted per PRB under a certain combination

Prob. of a combination.

Sum of all probabilities of each combination is equal to 1.
In order to model the different TD schedulers, a procedure is required to choose the $\eta$ users.
Infinitesimal Generator Matrix

\[ Q \]

\[
\begin{align*}
&0,0 & 0,1 & 0,2 & \cdots & 0,N_2 \\
&1,0 & 1,1 & 1,2 & \cdots & 1,N_2 \\
&N_1,0 & N_1,1 & N_1,2 & \cdots & N_1,N_2 \\
\end{align*}
\]

\[
\begin{align*}
&\mu_1(1,0) & \mu_1(1,1) & \mu_1(1,2) & \cdots & \mu_1(1,N_2) \\
&\mu_1(2,0) & \mu_1(2,1) & \mu_1(2,2) & \cdots & \mu_1(2,N_2) \\
&\mu_2(N_1,0) & \mu_2(N_1,1) & \mu_2(N_1,2) & \cdots & \mu_2(N_1,N_2) \\
\end{align*}
\]

\[
\begin{align*}
&N_2\lambda_2 & (N_2-1)\lambda_2 & (N_2-2)\lambda_2 & \cdots & \lambda_2 \\
&\lambda_1 & \lambda_1 & \lambda_1 & \cdots & \lambda_1 \\
&\lambda_1 & \lambda_1 & \lambda_1 & \cdots & \lambda_1 \\
\end{align*}
\]

\[
\begin{align*}
&\mu_2(N_1,1) & \mu_2(N_1,2) & \mu_2(N_1,3) & \cdots & \mu_2(N_1,N_2) \\
&\mu_2(N_1,1) & \mu_2(N_1,2) & \mu_2(N_1,3) & \cdots & \mu_2(N_1,N_2) \\
&\mu_2(N_1,1) & \mu_2(N_1,2) & \mu_2(N_1,3) & \cdots & \mu_2(N_1,N_2) \\
\end{align*}
\]
**Q Infinitesimal Generator Matrix**

\[
Q(i, i) = -\sum_j Q(i, j)
\]
MaxT: State Probability

MaxT Markov Chain State Probability (10UEs 5MB scenario)

MaxT Markov Chain State Probability (10UEs 10MB scenario)

MaxT Markov Chain State Probability (10UEs 15MB scenario)
BET: State Probability

BET Markov Chain State Probability (10UEs 5MB scenario)

BET Markov Chain State Probability (10UEs 10MB scenario)

BET Markov Chain State Probability (10UEs 15MB scenario)
MaxT: FTP Download Time

![Graph showing MaxT Average FTP download time (20UEs scenario)]

- **MaxT Average FTP download time (20UEs scenario)**
- Legend:
  - 5MB sim
  - 5MB analy
  - 10MB sim
  - 10MB analy
  - 15MB sim
  - 15MB analy

**Iner-arrival-time IAT (sec)**

**Download time (sec)**

- **5MB sim**
- **5MB analy**
- **10MB sim**
- **10MB analy**
- **15MB sim**
- **15MB analy**
MaxT: State Probability

MaxT Markov Chain State Probability (20UEs 5MB scenario)

MaxT Markov Chain State Probability (20UEs 10MB scenario)

MaxT Markov Chain State Probability (20UEs 15MB scenario)
Analysis 1: State Probability

Class 1 state \( i = 0 \)

Class 1 state \( i = 1 \)

Class 1 state \( i = 2 \)

Class 1 state \( i = 3 \)
Analysis 2: State Probability

Class 1 state \(i=0\)

Class 1 state \(i=1\)

Class 1 state \(i=2\)

Class 1 state \(i=3\)

Probability

IAT 15 sec sim
IAT 15 sec analy
IAT 30 sec sim
IAT 30 sec analy
IAT 45 sec sim
IAT 45 sec analy
IAT 60 sec sim
IAT 60 sec analy