# Future Mobile Communications:

## LTE Radio Scheduler Analytical Modeling

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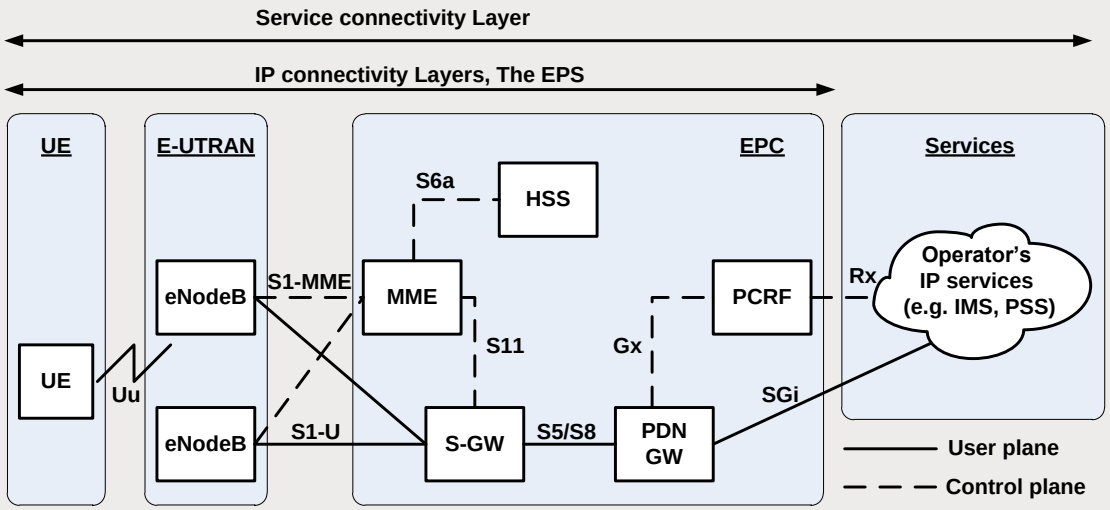
# Overview

# Long Term Evolution (LTE)

LTE is the newest 3GPP[1] standard (Release 8)

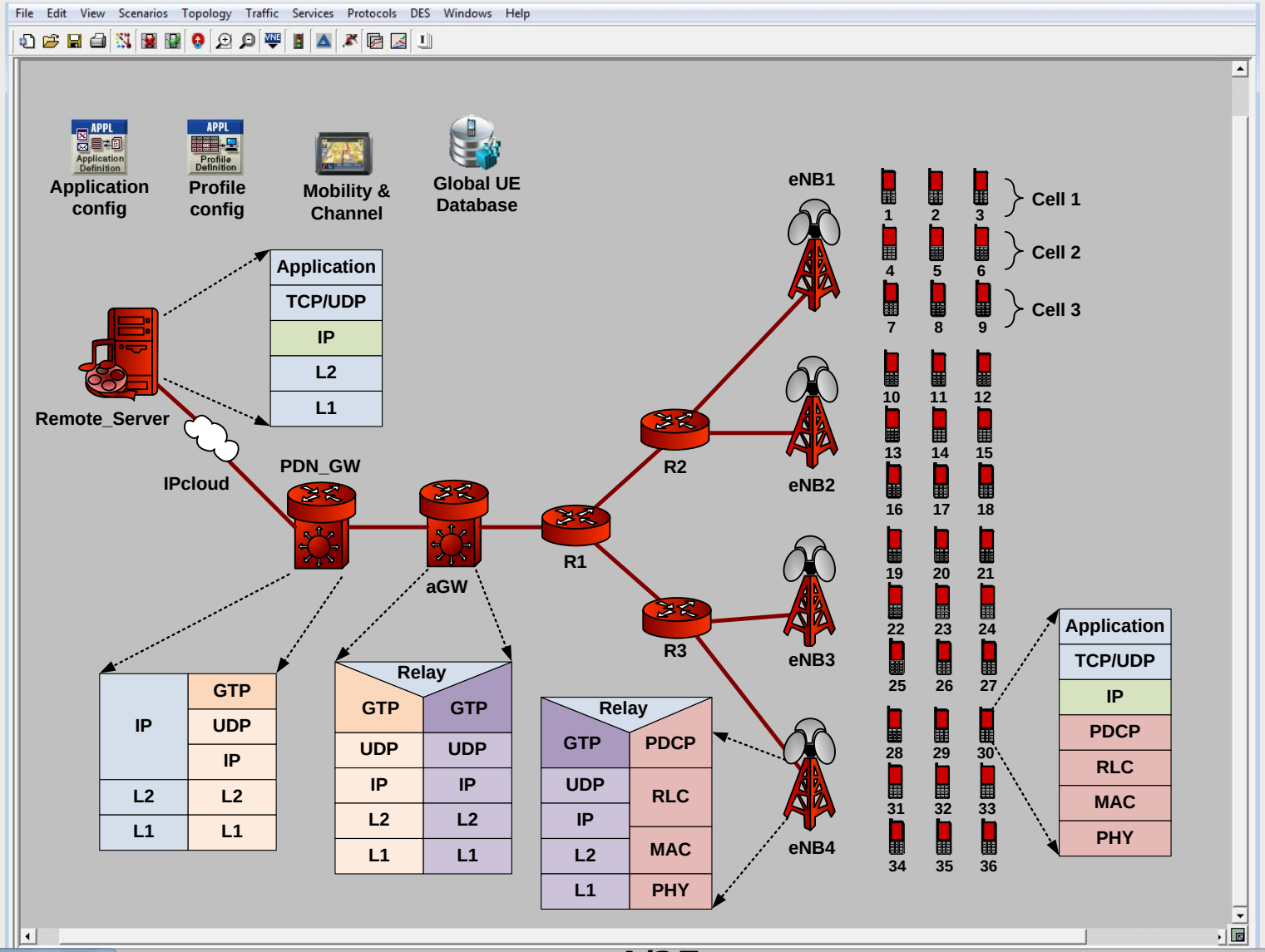



The main motivation of the work presented here is the analytical modeling of the LTE QoS aware radio scheduler.
And to validate our simulation results by comparing it to the analytical results.



[1] 3rd Generation Partnership Project

# LTE OPNET Simulation Model



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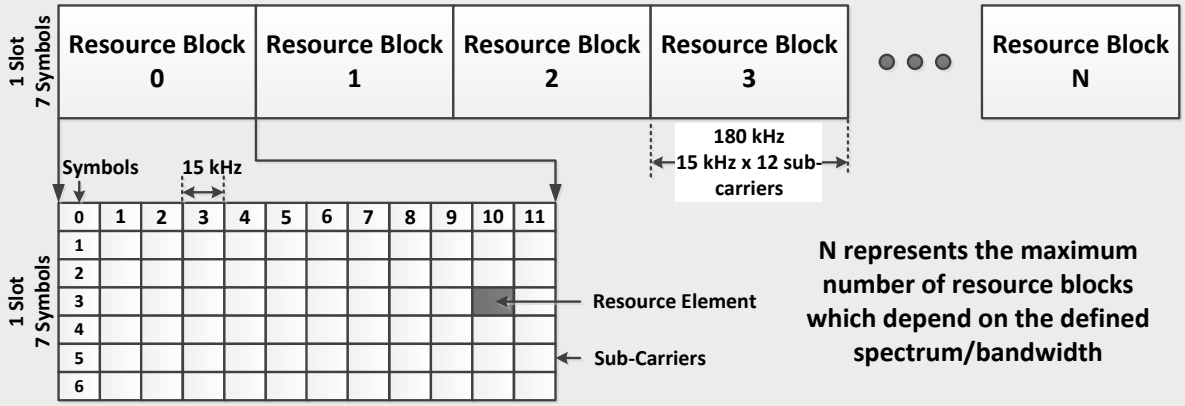
# LTE Physical Resource Structure

- Resources in LTE consist of both: time and frequency dimensions
- The smallest resource the scheduler can allocate is called a Physical Resource Block (PRB).
- A PRB consists of the following:
  - $7^2$ OFDM symbols in the time domain
  - 12 sub-carriers in the frequency domain

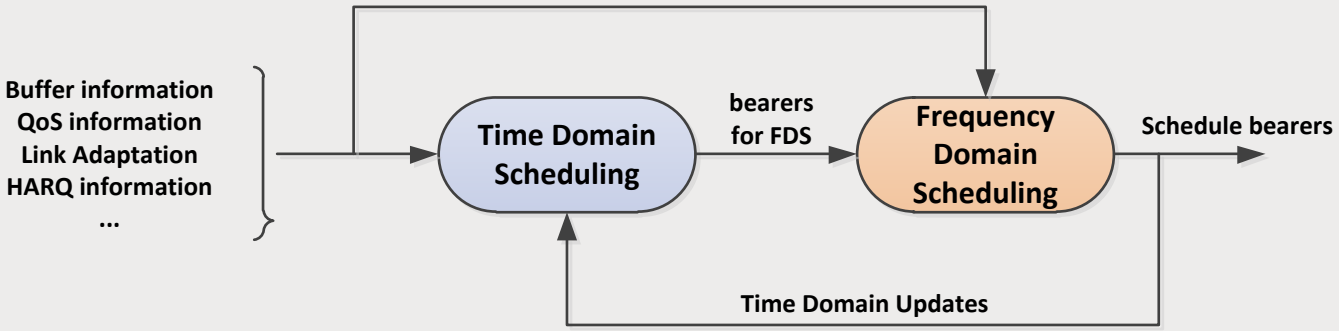





[2] 6 or 7 symbols depending on the length of the cyclic prefix

# LTE Dynamic Packet Scheduling

- The packet scheduling is divided into two different stages:
  - **Time Domain Scheduler (TDS)**: deals with QoS requirements and user/bearer prioritization
  - **Frequency Domain Scheduler (FDS)**: deals with spectrum allocation and multi-user diversity exploitation




- In LTE literature such a split is called *decoupled time and frequency domain scheduler*

# Time Domain Scheduler TDS

Two of the classical TDS schedulers are:

- **Blind Equal Throughput (BET)**: give a fair chance of resources so that users can achieve similar throughput
- **Maximum Throughput (MaxT)**: maximize the cell/system throughput by scheduling users with the best channel conditions

The TDS prioritization is done by calculating the priority factor.

$$P_k^{BET}(t) = argmax_k \left[ \frac{1}{\overline{\theta_k[t]}} \right] = \text{BET TD priority factor}$$

$$P_k^{MaxT}(t) = argmax_k \left[ SINR_k[t] \right] = \text{MaxT TD priority factor}$$

- $\overline{\theta_k[t]}$ is the normalized average throughput of bearer k ranging between 0 and 1
- $SINR_k[t]$ is the instantaneous SINR value of bearer k

# Frequency Domain Scheduler FDS

- Distributes the radio resources (PRBs) among the highest priority bearers obtained from the TDS
- Exploits the multi-user diversity and tries to enhance the overall spectral efficiency
- The FDS used in this work is an optimized round robin scheduler
  - Serves first strictly the GBR[3] bearers
  - Serves the highest $\psi$[4] nonGBR priority bearers with the remaining resources
  - Schedules the PRBs using an iterative approach



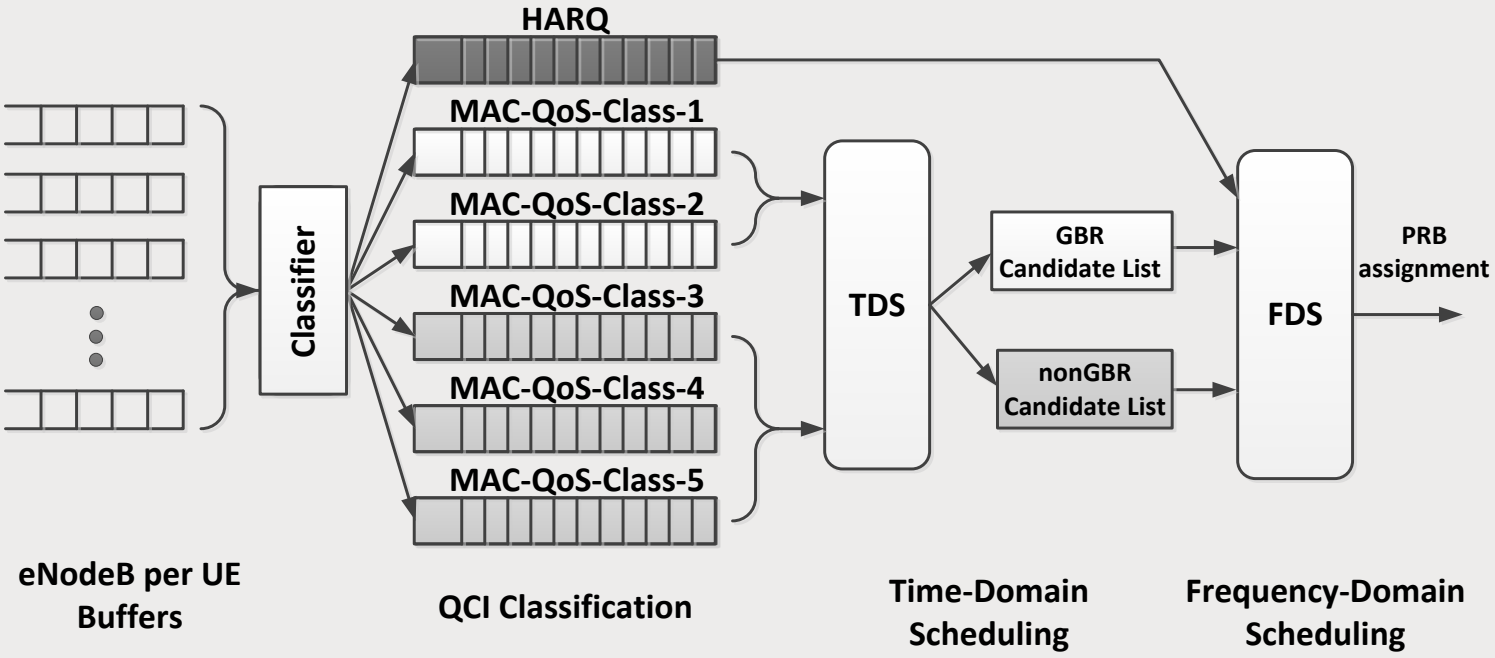
[3] Guaranteed Bit Rate

[4] $\psi$ is chosen to be 5 in this work

# Optimized Service Aware Scheduler

The proposed OSA scheduler provides:

- QoS guarantees
- Fairness among users
- System performance maximization



# OSA Scheduler: TDS and FDS

- The **OSA TDS** priority factor is:

$$P_k^{nonGBR-OSA}(t) = argmax_k \left[ W_{QoS\_j} \times \frac{\overline{\gamma_k[t]}}{\overline{\theta_k[t]}} \right]$$

  - $W_{QoS\_j}$ is the QoS weight of the $j^{th}$ QoS class
  - $\overline{\theta_k[t]}$ is the normalized EMA[5] throughput of bearer k
  - $\overline{\gamma_k[t]}$ is the normalized EMA channel condition of bearer k

- The OSA serves the GBR bearers (i.e., VoIP) with strict priority before the non-GBR bearers
- The **OSA FDS** is an optimized round robin scheduler



[5] Exponential Moving Average

# LTE Scheduler Analytical Models

- The presented model is an extension of the model found in [1]
- The scheduler has a fixed amount of PRBs to distribute among active UEs every TTI [6]
- The UEs have different SINR
  - Support different Modulation and Coding Schemes (MCS)

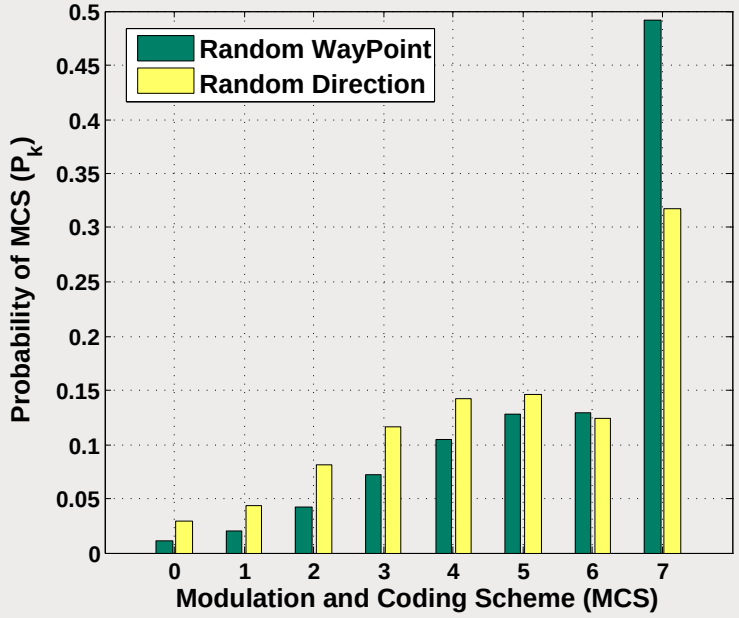
[1] S. Doirieux, ... "An efficient analytical model for the dimensioning of WiMAX networks supporting multi-profile best effort traffic", Computer Communications, Volume 33, Issue 10, 15 June 2010



[6] Transmission Time Interval = 1ms

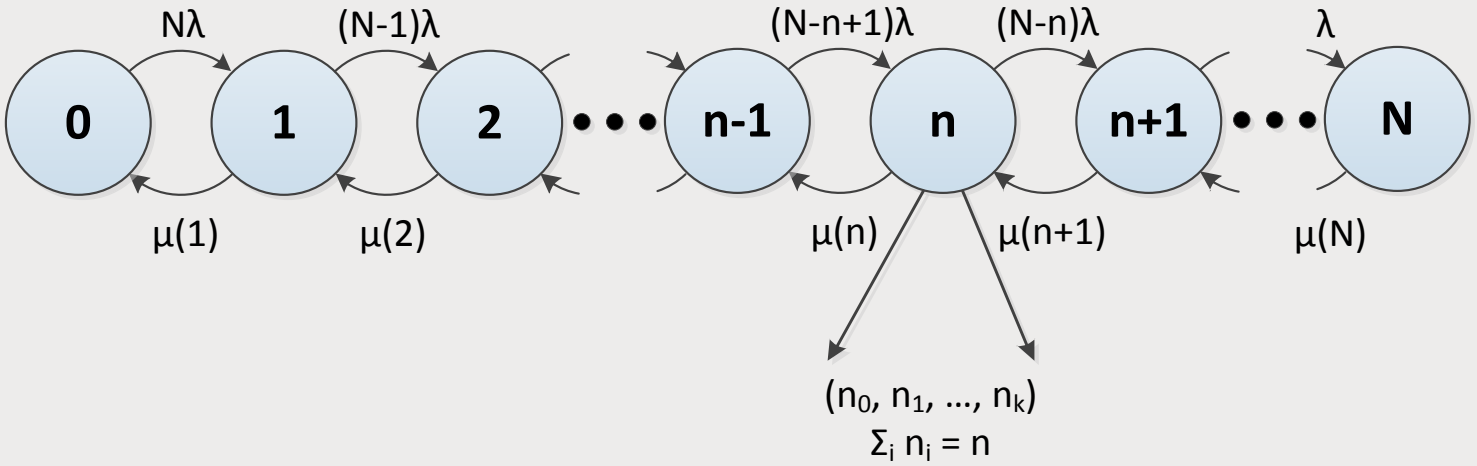
# Model Assumptions

- UEs are statistically identical with respect to the channel
- The channel is divided into 8 states ($MCS_k$) ($0 < k < 7$)
- Each $MCS_k$ has a static probability $P_k$ (Mobility model dependent)
- UEs use the same ON/OFF elastic traffic:
  - $\overline{X_{ON}}$ is the average file size in bits
  - $\overline{t_{off}}$ is the average OFF duration in s



# General Analytical Model

- The model is based on the Continuous Time Markov Chain (CTMC)
  - n: Markov chain state representing the number of active UEs in a TTI
  - N: total number of UEs in the system
- The resulting CTMC consists of N+1 states
  - Arrival transition is performed with rate $(N-n)\lambda$ with $\lambda = 1/\overline{t_{off}}$
  - Departure transition is performed with a generic rate $\mu(n)$



# LTE Frequency Domain Scheduler

- The FDS is a round robin scheduler
- It serves up to $\psi$=5 users per TTI
- This means, that the UEs served per TTI are equal to $\eta$:

$$\eta = min[n - n_0, \psi]$$

with n being the number of active users in a TTI, and $n_0$ is the number of users in outage[7]



[7] User is in bad channel condition that does not allow any transmission

# Steady State Probability

- The steady state probability $\pi(n)$ can be calculated from:

$$\boldsymbol{\pi} \cdot \mathbf{Q} = 0 \tag{1}$$

  - $\boldsymbol{\pi}$=$[\pi_0, \pi_1, ...\pi_N]$ and $\sum_i \pi_i$ = 1
  - **Q** is the infinitesimal generator matrix

- According to Little's law, the average ON period duration $\overline{t_{ON}}$[8] is:

$$\overline{t_{on}} = \frac{\sum_{n=1}^{N} n\pi(n)}{\sum_{n=1}^{N}(N-n+1)\lambda \cdot \pi(n)}$$



[8] $\overline{t_{ON}}$ is the duration of an active transfer, or file download time

# Departure Rate $\mu(n)$

$$\mu(n) = \frac{\sum_{\substack{(n_0,\ldots,n_K)=(0,\ldots,0)| \\ n_0+\ldots+n_K=n \\ n_0\neq n}}^{(n,\ldots,n)} \overline{TBS}(n_0,\ldots,n_K) \binom{n}{n_0,\ldots,n_K} \left[\prod_{k=0}^{K} P_k^{n_k}\right]}{\overline{X_{on}} \cdot TTI}$$

- $\overline{X_{on}}$ is the average file size of the ON period
- $\binom{n}{n_0,\ldots,n_K}$ is the multinomial coefficient
- $P_k$ is the stationary probability of $MCS_k$
- $n_k$ is the number of active users in $MCS_k$
- $\overline{TBS}(n_0,\ldots,n_K)$ is the total number of bits transmitted for all served users under a certain combination $(n_0,\ldots,n_K)$

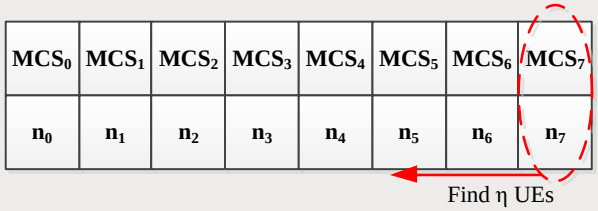
# $\overline{TBS}(n_0, \ldots, n_K)$

$$\overline{TBS}(n_0, \ldots, n_K) = \sum_{i=[i_1,\ldots,i_\eta]} TBS_i(\eta) \tag{2}$$

- $\eta$ is the number of users served under $(n_0, \ldots, n_K)$
- $[i_1, \ldots, i_\eta]$ represents the index of the chosen $\eta$ users out of the n active users considered for scheduling within a TTI
- $TBS_i(\eta)$ denotes the number of bits of user i using his $MCS$
- the choice of the $\eta$ users is determined by the TD scheduler.

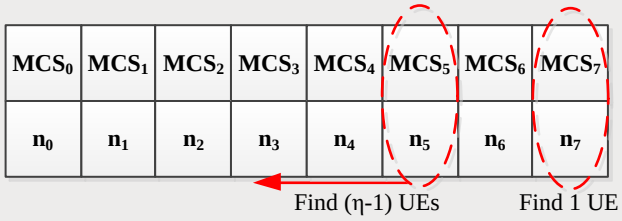
# Choosing $\eta$ UEs Procedure

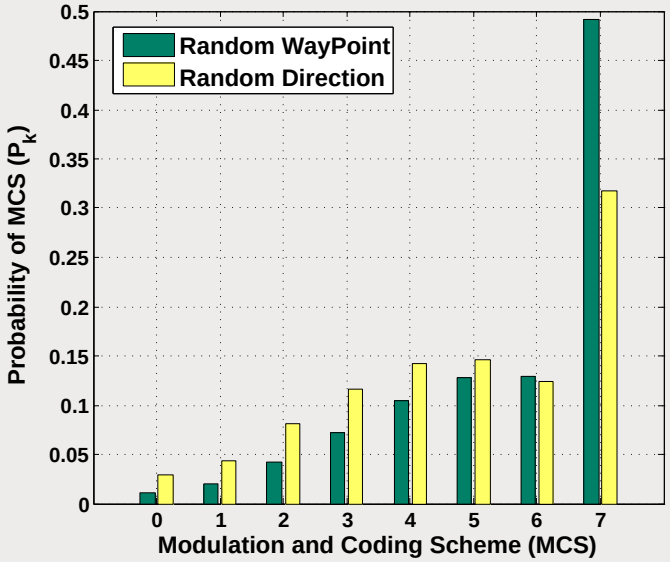
- MaxT chooses the $\eta$ UEs with the highest MCS



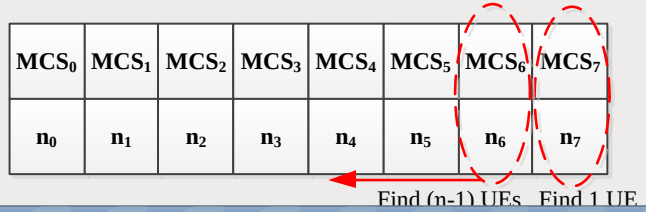

$$AverageThroughput = \sum_{k=1}^{K} TBS_k \cdot P_k$$

- BET chooses the $\eta$ UEs from the MCS that achieves the avg. throughput:



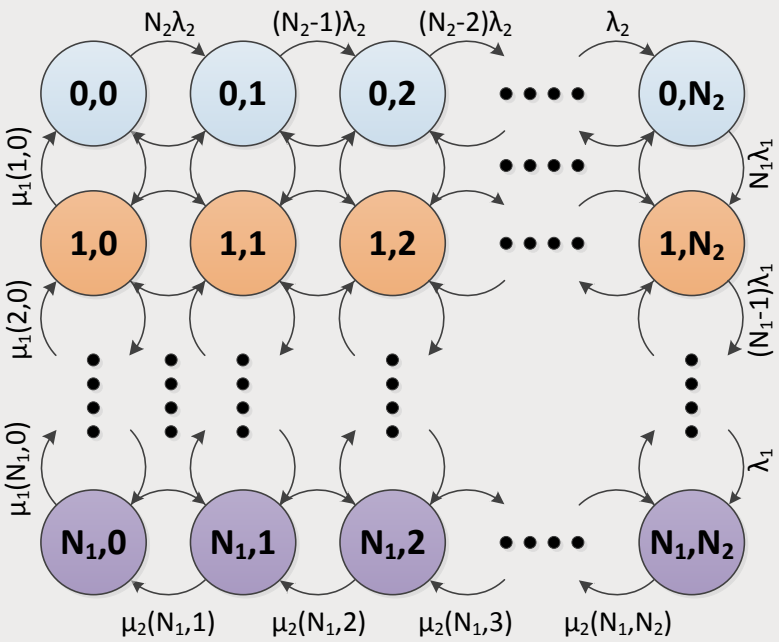




- OSA chooses the $\eta$ UEs from the MCS as follows:



# Two Classes Model

- The single class model is extended into a two dimensional Markov Chain, with state (i,j)
  - i represents the number of UEs within class 1
  - j represents the number of UEs within class 2
- The combinations within each state will increase to:
  $(i_0, i_1, i_2, i_3, i_4, i_5, i_6, i_7, j_0, j_1, j_2, j_3, j_4, j_5, j_6, j_7)$
  where $\sum_r i_r = N_1$ and = $\sum_r j_r = N_2$



# Three Classes Model

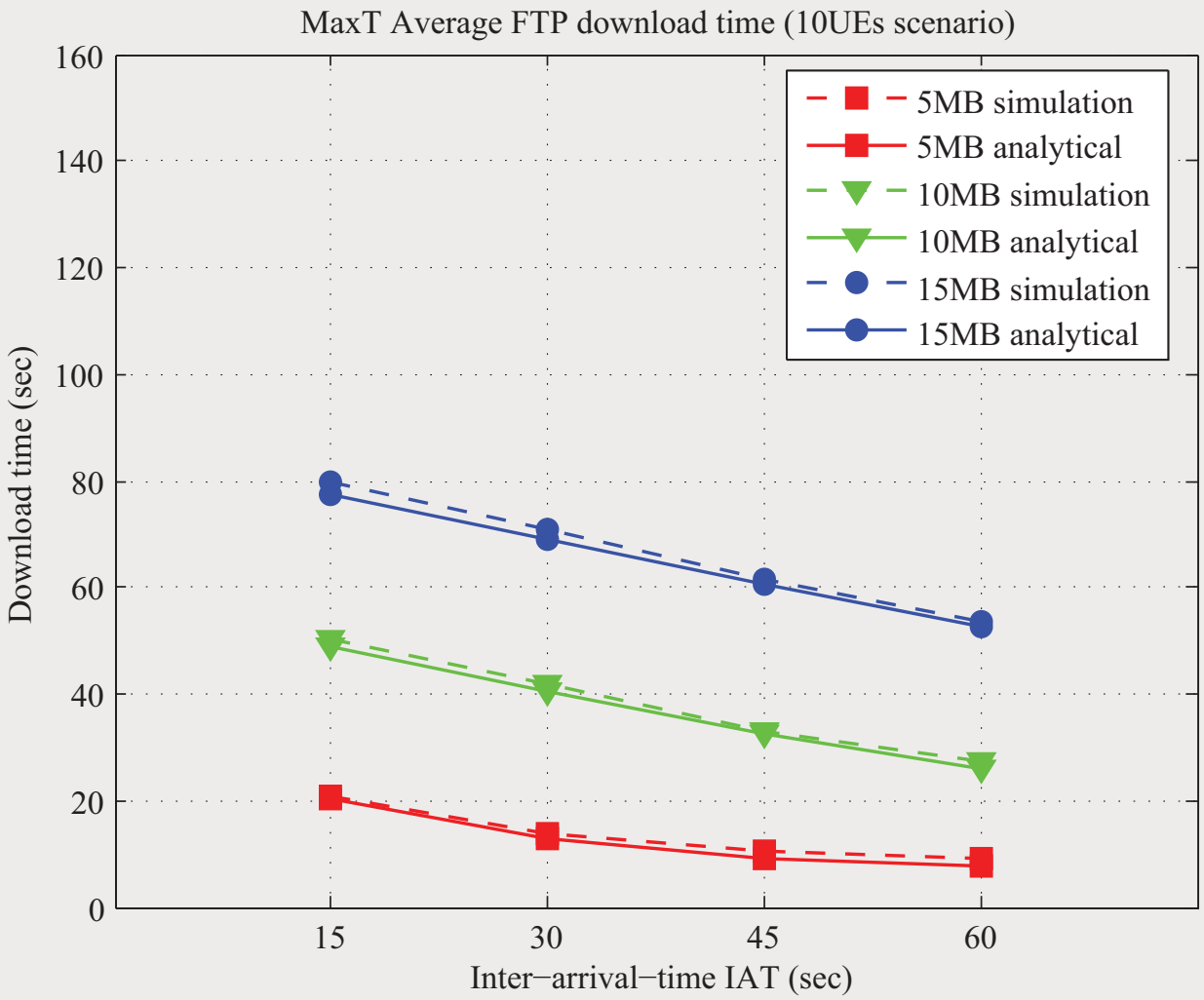
- Similary the single class model can be extended into a three dimensional Markov Chain, with state (i,j,z)
  - i represents the number of UEs within class 1
  - j represents the number of UEs within class 2
  - z represents the number of UEs within class 3
- The combinations within each state will increase to:
  $(i_0, i_1, i_2, i_3, i_4, i_5, i_6, i_7, j_0, j_1, j_2, j_3, j_4, j_5, j_6, j_7, z_0, z_1, z_2, z_3, z_4, z_5, z_6, z_7)$
  where $\sum_r i_r = N_1$ and $\sum_r j_r = N_2$
  and $\sum_r z_r = N_3$

# Scenarios Configuration

| General parameter | Value |
|---|---|
| Bandwidth (# PRBs) | 5 MHz (i.e., 25 PRBs) |
| Radio QoS weights | $W_{QoS1}$=5, $W_{QoS2}$=2 and $W_{QoS3}$=1 |
| Traffic model | FTP with different IATs: |
| | uniform (0, 30) s, uniform (15, 45) s, |
| | uniform (30, 60) s and uniform (45, 75) s |
| Simulation run time | 8000 s, 10 seeds with 95% confidence interval |

| Analysis | Class1 | Class2 | Class3 |
|---|---|---|---|
| MaxT | 10 UEs with | | |
| | 5, 10 and 15 MByte | | |
| | 3 UEs with | 6 UEs with | |
| w-MaxT | 5 MByte | 10 MByte | |
| | 3 UEs with | 3 UEs with | 3 UEs with |
| w-MaxT | 5 MByte | 10 MByte | 15 MByte |

# Average FTP Download Time: MaxT



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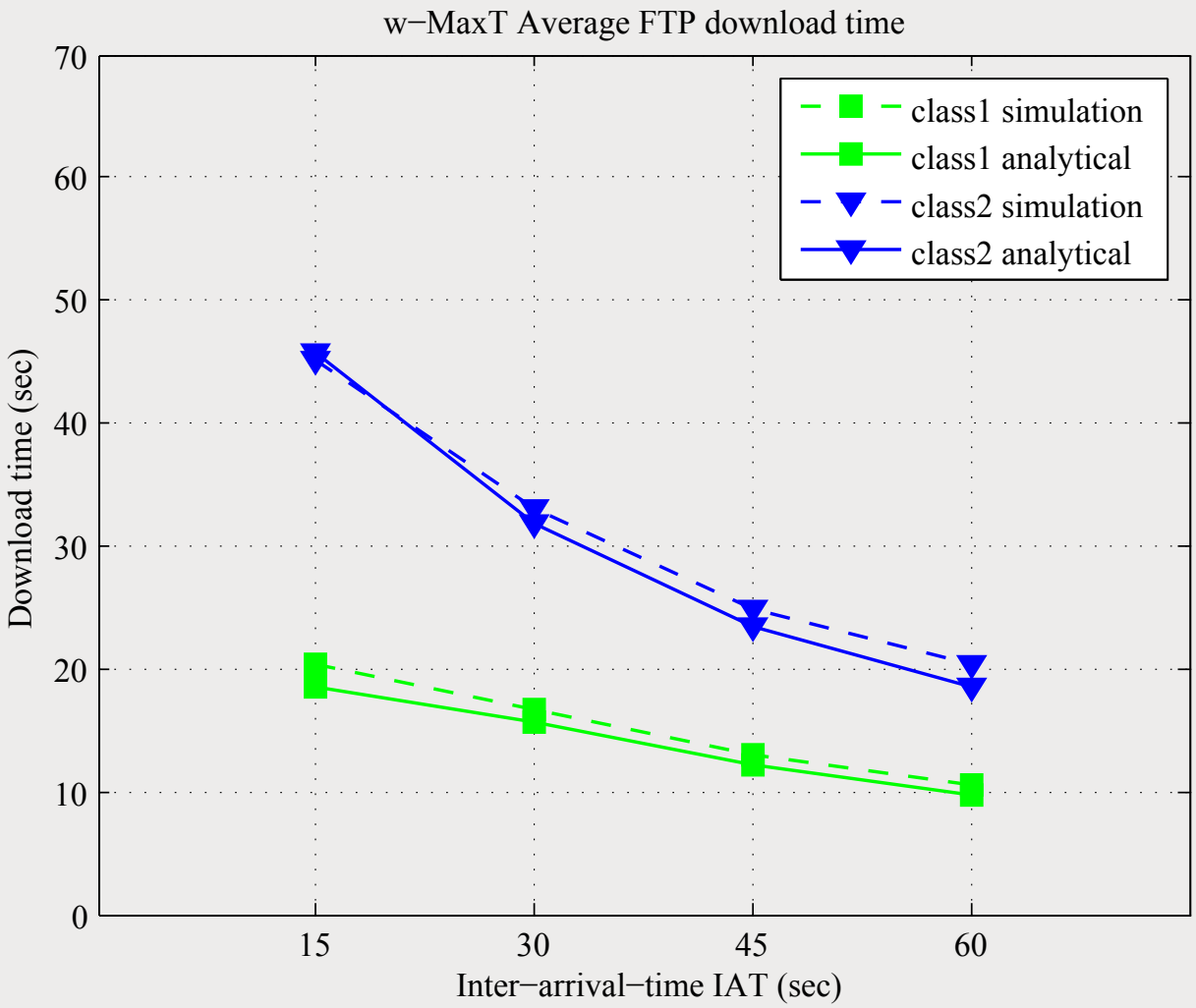
# Average FTP Download Time: w-MaxT



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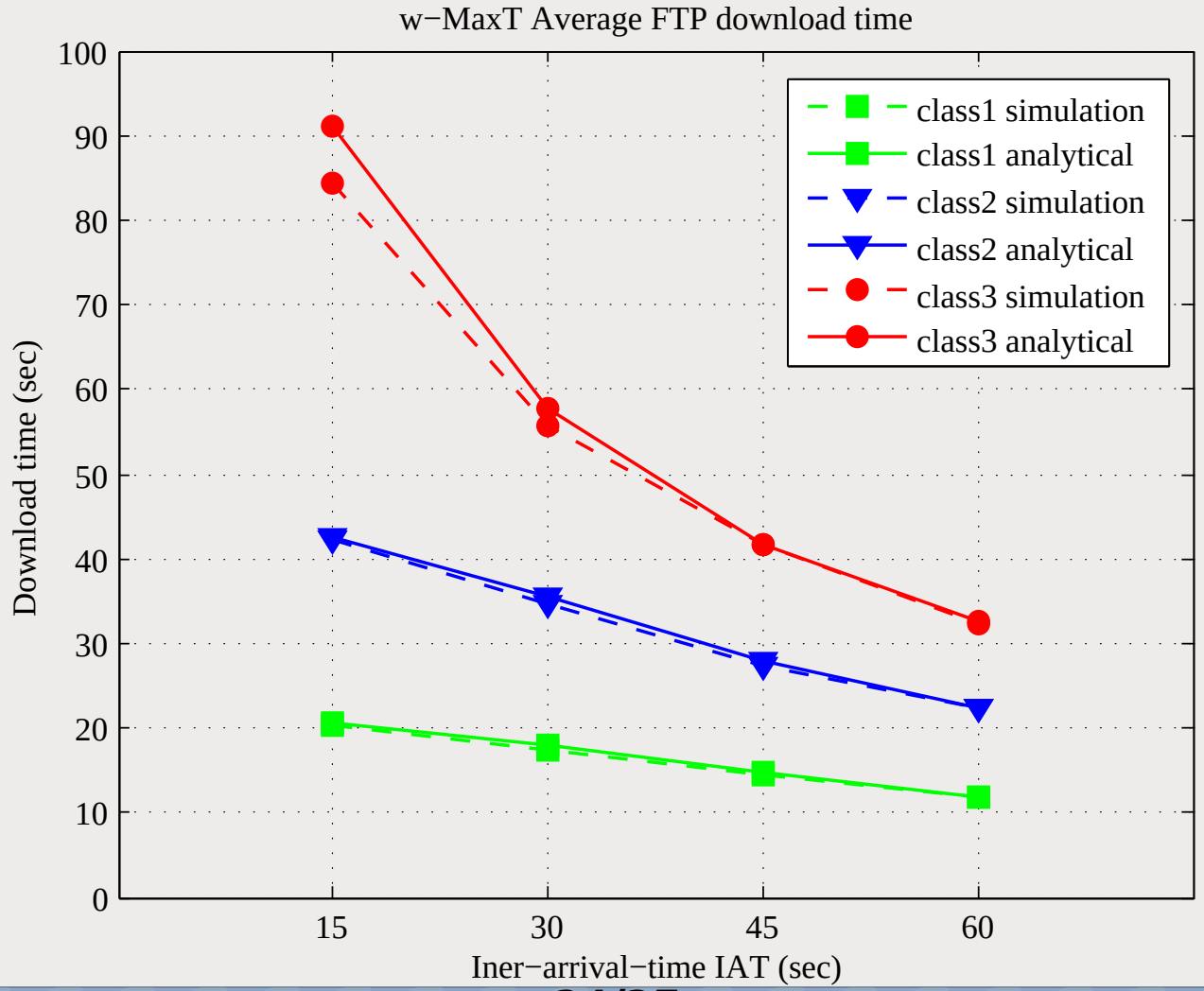
# Average FTP Download Time: w-MaxT



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# Conclusion and Outlook

- The proposed analytical model shows very accurate results compared to the simulation results
- The model can support up to three different QoS classes
- The analytical model only requires a couple of minutes to run (much faster than simulations)
- The model can be extended to:
  - support more QoS classes
  - a GBR class can also be modeled
  - other types of TDS can also be modeled

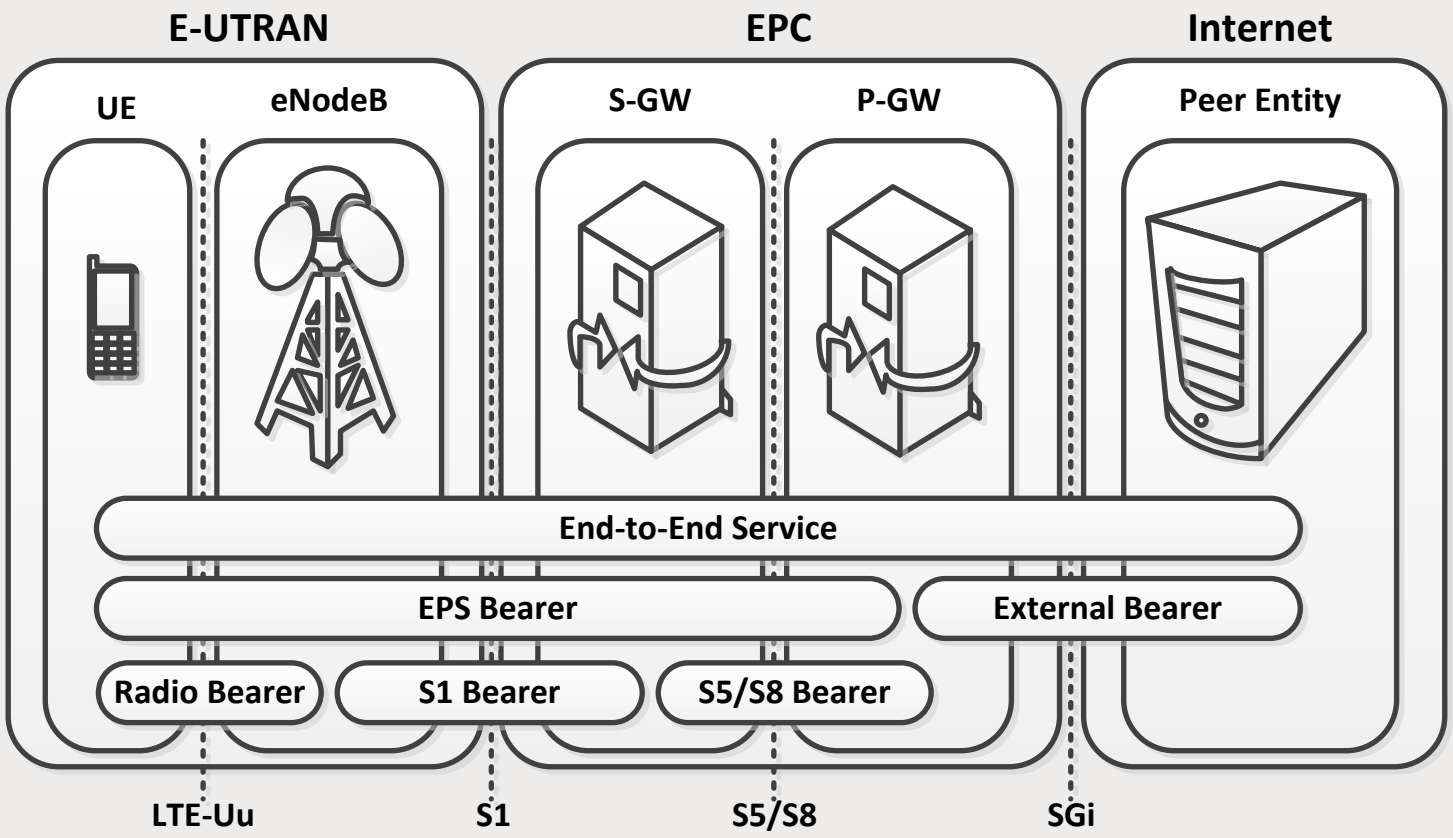
# Thanks for Listening

# Backup issues

- LTE QoS bearers
- 3GPP QoS bearer classification
- Channel dependent scheduling
- Channel model
- Analytical model scaling
- Analytical model TBS(n)
- Analytical model Q-matrix

# BACKUP

# 3GPP QoS Bearer Classification

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# LTE QoS Bearers

| QCI | Bearer type | Priority | Packet delay budget (ms) | Packet error loss rate | Example services |
|---|---|---|---|---|---|
| 1 | GBR | 2 | 100 | $10^{-2}$ | Conversational voice |
| 2 | GBR | 4 | 150 | $10^{-3}$ | Conversational video (live streaming) |
| 3 | GBR | 5 | 300 | $10^{-6}$ | Non-conversational video (buffered streaming) |
| 4 | GBR | 3 | 50 | $10^{-3}$ | Real time gaming |
| 5 | non-GBR | 1 | 100 | $10^{-6}$ | IMS signaling |
| 6 | non-GBR | 7 | 100 | $10^{-3}$ | Voice, video (live streaming), interactive gaming |
| 7 | non-GBR | 6 | 300 | $10^{-6}$ | Video (buffered streaming) |
| 8 | non-GBR | 8 | 300 | $10^{-6}$ | TCP based (e.g., www, e-mail, chat, FTP, p2p) |
| 9 | non-GBR | 9 | 300 | $10^{-6}$ | |

- Nine predefined QoS classes
  - four Guaranteed Bit Rate (GBR)
  - five non Guaranteed Bit Rate (nonGBR)
- Operators are free to choose their own classification

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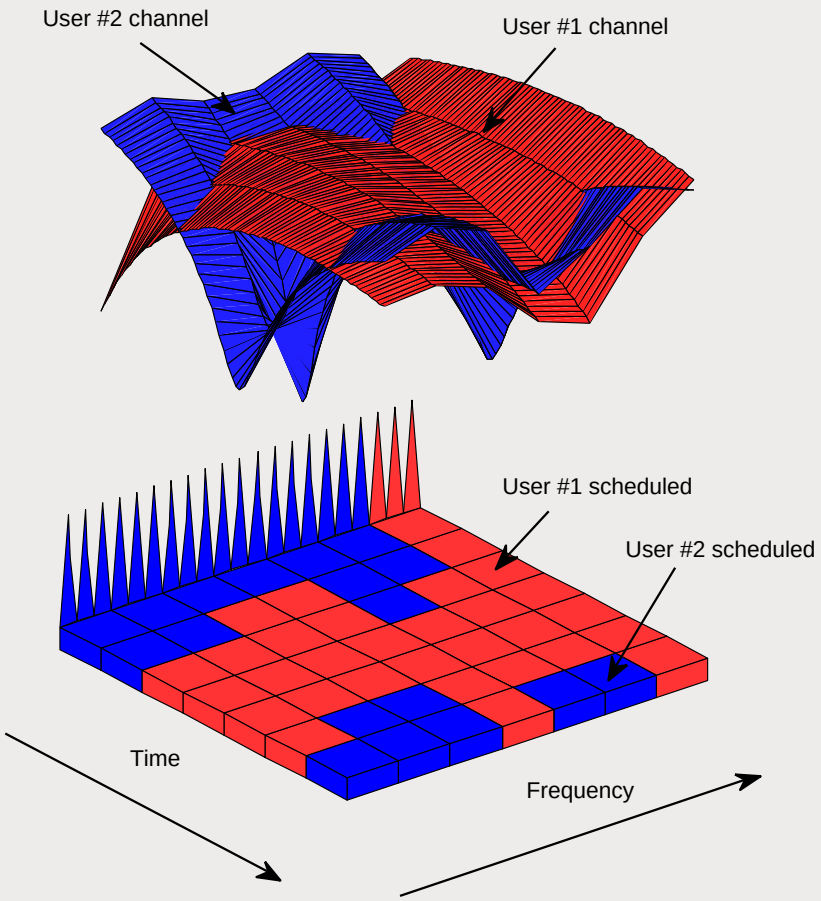
# Channel Dependent Scheduling

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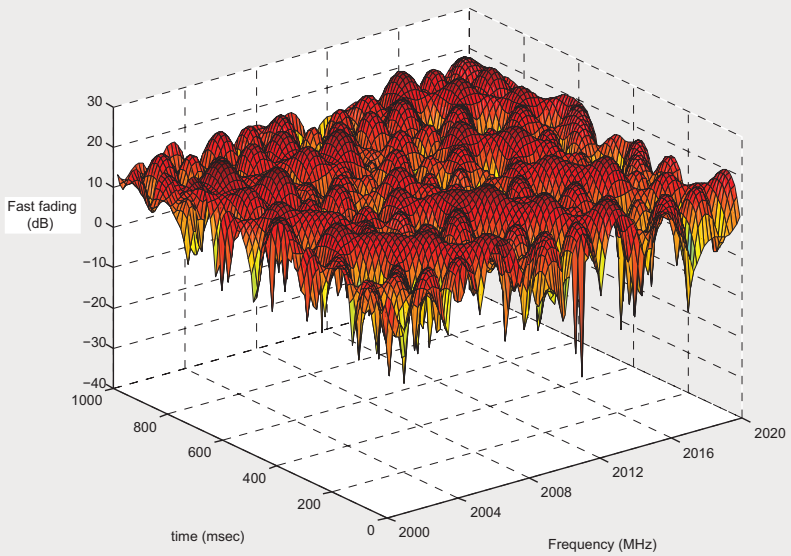
# Channel Model

- **Path loss**:

$$128.1 + 37.6 \cdot log_{10}(distance)$$

- **Correlated slow fading**:
  - Log-normal distributed
- **Fast fading**:
  - Jakes-like method
  - Two extended ITU channel models
    - Extended pedestrian A
    - Extended vehicular A

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# Steady State Probability

- Another solution wiil be to use the closed loop solution from the birth-and-death process of the Markov chain:

$$\pi(n) = \left[ \prod_{i+1}^{n} \frac{(N - i + 1)\lambda}{\mu(i)} \right] \pi(0)$$

- $\pi(0)$ is obtained by normalization

back

# $\overline{TBS}(2)$

- if we have two active UEs i.e. n=2
- those two users can be at any MCS (from $MCS_0$ to $MCS_7$)
- if we represent the number of users in each $MCS_k$ by $n_k$ we get: $(n_0, n_1, n_2, n_3, n_4, n_5, n_6, n_7)$ with $n_0 + ... + n_7 = n$
- then we have the following possible combinations

| | MCS0 | MCS1 | MCS2 | MCS3 | MCS4 | MCS5 | MCS6 | MCS7 |
|---|---|---|---|---|---|---|---|---|
| Comb1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Comb2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| Comb3 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| Comb4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| Comb5 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| Comb6 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| Comb7 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| Comb8 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| Comb9 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| Comb10 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| Comb11 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| Comb12 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| Comb13 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| Comb14 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| Comb15 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| Comb16 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Comb17 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| Comb18 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Comb19 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| Comb20 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| Comb21 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| Comb22 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| Comb23 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| Comb24 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| Comb25 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| Comb26 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Comb27 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Comb28 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| Comb29 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Comb30 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Comb31 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Comb32 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Comb33 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Comb34 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Comb35 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

back

# $\overline{TBS}(2)$

$$\overline{TBS}(2) = \sum_{\substack{(n_0,\ldots,n_K)=(0,\ldots,0)| \\ n_0+\ldots+n_K=0 \\ n_0\neq 2}}^{(2,\ldots,2)} \overline{TBS}(n_0,\ldots,n_K) \binom{2}{n_0,\ldots,n_K} \left[\prod_{k=0}^{K} P_k^{n_k}\right] \tag{3}$$

| | MCS0 | MCS1 | MCS2 | MCS3 | MCS4 | MCS5 | MCS6 | MCS7 |
|---|---|---|---|---|---|---|---|---|
| Comb1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 |
| Comb2 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| Comb3 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 0 |
| Comb4 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| Comb5 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| Comb6 | 0 | 0 | 0 | 0 | 0 | 2 | 0 | 0 |
| Comb7 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| Comb8 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 0 |
| Comb9 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| Comb10 | 0 | 0 | 0 | 0 | 2 | 0 | 0 | 0 |
| Comb11 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| Comb12 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 |
| Comb13 | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 |
| Comb14 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| Comb15 | 0 | 0 | 0 | 2 | 0 | 0 | 0 | 0 |
| Comb16 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Comb17 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| Comb18 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Comb19 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 |
| Comb20 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| Comb21 | 0 | 0 | 2 | 0 | 0 | 0 | 0 | 0 |
| Comb22 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| Comb23 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 0 |
| Comb24 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| Comb25 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 |
| Comb26 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| Comb27 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| Comb28 | 0 | 2 | 0 | 0 | 0 | 0 | 0 | 0 |
| Comb29 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| Comb30 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| Comb31 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| Comb32 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| Comb33 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| Comb34 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Comb35 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |

Average number of bits transmitted per PRB under a certain combination

Prob. of a combination.

Sum of all probabilities of each combination is equal to 1.

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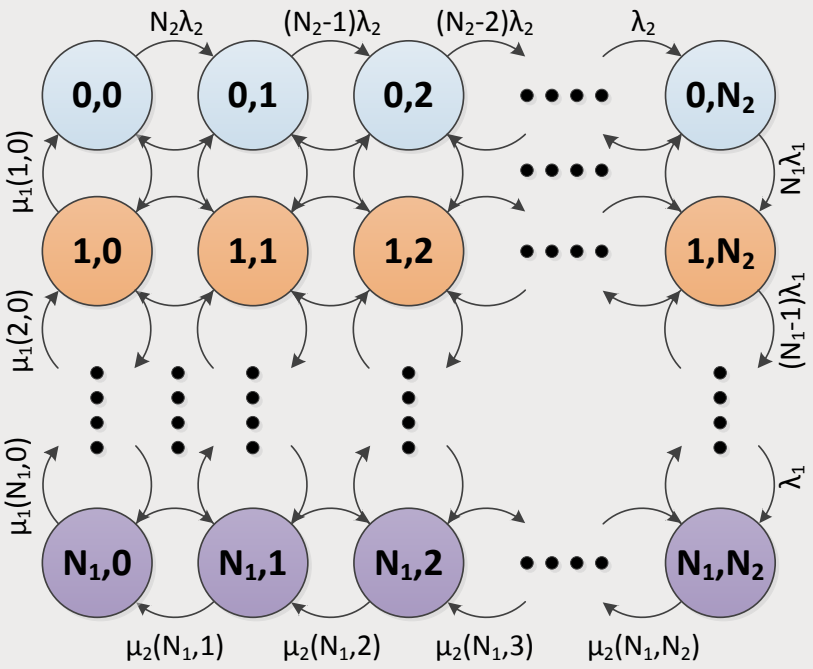
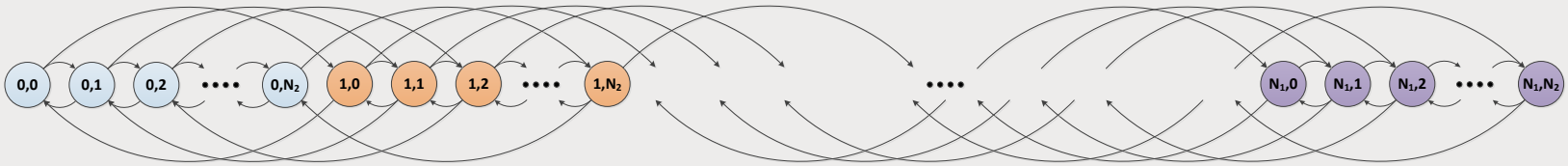
# Q Infinitesimal Generator Matrix

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# Q Infinitesimal Generator Matrix

**Markov chain state (i,j)**

| Markov chain state (i,j) | (0,0) | (0,1) | (0,2) | (0,3) | (0,4) | (1,0) | (1,1) | (1,2) | (1,3) | (1,4) | (2,0) | (2,1) | (2,2) | (2,3) | (2,4) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (0,0) | | 4λ1 | | | | 2λ2 | | | | | | | | | |
| (0,1) | $\mu_1$ (0,1) | | 3λ1 | | | | 2λ2 | | | | | | | | |
| (0,2) | | $\mu_1$ (0,2) | | 2λ1 | | | | 2λ2 | | | | | | | |
| (0,3) | | | $\mu_1$ (0,3) | | λ1 | | | | 2λ2 | | | | | | |
| (0,4) | | | | $\mu_1$ (0,4) | | 0 | | | | 2λ2 | | | | | |
| (1,0) | $\mu_2$ (1,0) | | | | $\mu_1$ (1,0) | | 4λ1 | | | | λ2 | | | | |
| (1,1) | | $\mu_2$ (1,1) | | | | $\mu_1$ (1,1) | | 3λ1 | | | | λ2 | | | |
| (1,2) | | | $\mu_2$ (1,2) | | | | $\mu_1$ (1,2) | | 2λ1 | | | | λ2 | | |
| (1,3) | | | | $\mu_2$ (1,3) | | | | $\mu_1$ (1,3) | | λ1 | | | | λ2 | |
| (1,4) | | | | | $\mu_2$ (1,4) | | | | $\mu_1$ (1,4) | | 0 | | | | λ2 |
| (2,0) | | | | | | $\mu_2$ (2,0) | | | | $\mu_1$ (2,0) | | 4λ1 | | | |
| (2,1) | | | | | | | $\mu_2$ (2,1) | | | | $\mu_1$ (2,1) | | 3λ1 | | |
| (2,2) | | | | | | | | $\mu_2$ (2,2) | | | | $\mu_1$ (2,2) | | 2λ1 | |
| (2,3) | | | | | | | | | $\mu_2$ (2,3) | | | | $\mu_1$ (2,3) | | λ1 |
| (2,4) | | | | | | | | | | $\mu_2$ (2,4) | | | | $\mu_1$ (2,4) | |

$$Q(i,i) = -\sum_{j} Q(i,j)$$

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# Performance Parameters

- The average number of active users $\overline{Q}$ is expressed as:

$$\overline{Q1} = \sum_{N1}^{i=0} \sum_{N2}^{j=0} i\pi(i,j)$$

- The mean number of departures by unit of time is $\overline{D}$ and is obtained as:

$$\overline{D1} = \sum_{N1}^{i=0} \sum_{N2}^{j=0} \mu_1(i,j)\pi(i,j)$$

- According to Little's law, the average ON period duration $\overline{t_{ON}}$ (i.e. duration of an active transfer):

$$\overline{t_{1on}} = \frac{\overline{Q1}}{\overline{D1}}$$

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# Departure Rate $\mu 1(i,j)$

$$\mu 1(i,j) = \frac{\overline{TBS1}(i,j)}{\overline{X1_{on}} \cdot TTI}$$

- $\overline{TBS1}(i,j)$ is the average amount of bits sent by all served UEs in a TTI
- $\overline{X1_{on}}$ is the average file size of class 1 UEs
- TTI is the Transmission Time Interval (1ms)

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# Generic Average Bit Rate $\overline{TBS\,1}(i,j)$

$$\overline{TBS\,1}(i,j) = \sum_{\substack{(i_0,\ldots,i_K,j_0,\ldots,j_K)=(0,\ldots,0)| \\ i_0+\ldots+i_K=i \\ j_0+\ldots+j_K=j \\ i_0\neq i \text{ with } j_0\neq j}}^{(i,\ldots,i,j,\ldots,j)} i!j!\overline{TBS\,1}(i_0,\ldots,i_K,j_0,\ldots,j_K)\left[\prod_{k=0}^{K}\frac{P_k^{i_k}P_k^{j_k}}{i_k!j_k!}\right]$$

- $P_k$ is the stationary probability of $\mathrm{MCS}_k$
- $i_k$ is the number of class 1 active users in $\mathrm{MCS}_k$
- $j_k$ is the number of class 2 active users in $\mathrm{MCS}_k$
- $\overline{TBS\,1}(i_0,\ldots,i_K,j_0,\ldots,j_K)$ is the total bits transmitted for all served UEs under a certain combination $(i_0,\ldots,i_K,j_0,\ldots,j_K)$

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# $\overline{TBS1}(i_0, \ldots, i_K, j_0, \ldots, j_K)$

$$\overline{TBS1}(i_0, \ldots, i_K, j_0, \ldots, j_K) = \sum_{r=[r_1,\ldots,r_{\delta 1}]} TBS_r(\eta)$$

$$r_{\delta 1} + r_{\delta 2} = \eta$$

- r is a vector representing the indices of the served users in class1
- $r_{\delta 1}$ is the number of users served in class1.
- $\overline{TBS2}(i_0, \ldots, i_K, j_0, \ldots, j_K)$ can be calculated similarly

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# choosing *eta* Users

$$AverageThroughput = \sum_{k=1}^{K} TBS_k \cdot P_k$$

- K is the number of MCSs
- $TBS_k$ is the transport block size using $MCS_k$
- $P_k$ is the static probability of $MCS_k$
- Avg. throughput for RWP is $MCS_5$
- Avg. throughput for RD is $MCS_4$

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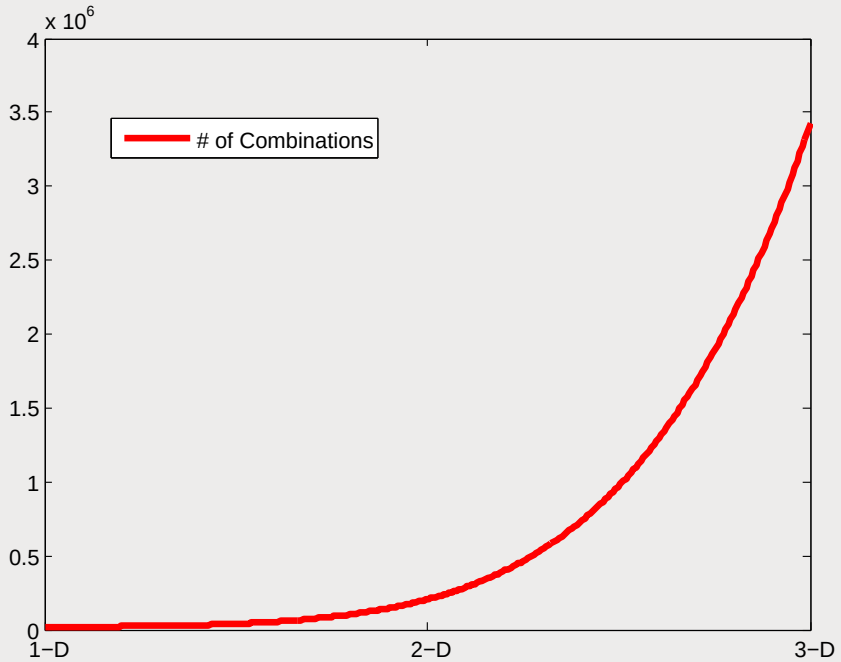
# Analytical Model Scaling

These measurements were performed in MATLAB using a server with AMD Phenom(tm) 9850 Quad-Core Processor 2.50 GHz, 8.00 GB of RAM, and 64-bit Windows 7 OS.

| Model | Number of UEs | Combinations | Run time |
|---|---|---|---|
| 1-D | 10 UEs | 19448 | 2 seconds |
| | 20 UEs | 888029 | 79 seconds |
| 2-D | (3,6) UEs | 205919 | 49 seconds |
| 3-D | (2,3,5) UEs | 3421440 | 13 minutes ?? |

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